Networks: Spring 2008	**Homework 1**
David Easley and Jon Kleinberg	**Due February 6, 2008**

Please hand in the solutions to the different parts separately, to facilitate grading.

Reading: The questions below are primarily based on the material in Chapters 1-3 of the Networks book draft.

Also, in writing the solutions to this problem set, we noted two corrections to Chapter 2. First, in both panels of Figures 2.1 and 2.2, there should not be an edge connecting node A to node F. Second, the paragraph on page 30 discussing Figure 2.5 should say that if the A-F edge were relabeled to be a strong tie rather than a weak tie, then neither node F nor node A would satisfy the Strong Triadic Closure Property after this relabeling.

Part A

(1) In the social network depicted in Figure 1, with each edge labeled as either a strong or weak tie, which nodes satisfy the Strong Triadic Closure Property from Chapter 2, and which do not? Provide an explanation for your answer.

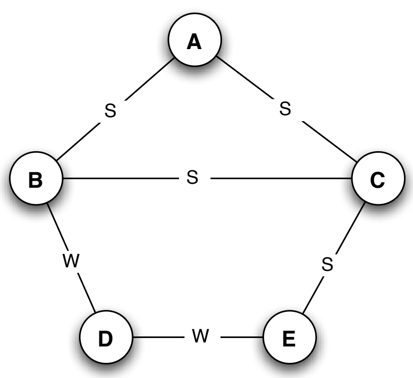

Figure 1: The graph for Question 1.

(2) Suppose some researchers are investigating the presence or absence of structural balance within a group of people, all pairs of whom know each other. Thus, the social network within the group can be represented, as in Chapter 3, by a labeled complete graph, in which each edge is labeled either + or −.

However, the researchers have only been able to observe certain pairs of people so as to determine the sign of the edge connecting them; for the remaining pairs, they do not know

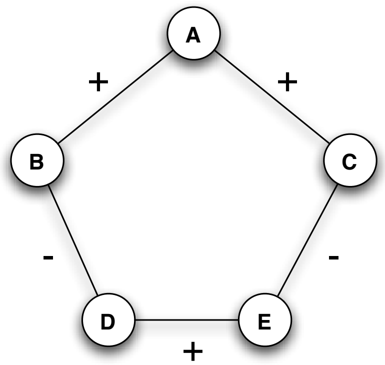

Figure 2: The graph for Question 2.

the sign. The data they have collected is shown in Figure 2. They want to know whether this partial data they have collected is consistent with structural balance or not.

Concretely, this leads to the following question: is it possible to fill in the missing labeled edges in Figure 2 so as to get a labeled complete graph that is balanced? If so, describe the labels you would put on the missing edges. If not, provide an explanation for why there is no way to fill in the missing edges so as to end up with a balanced complete graph.

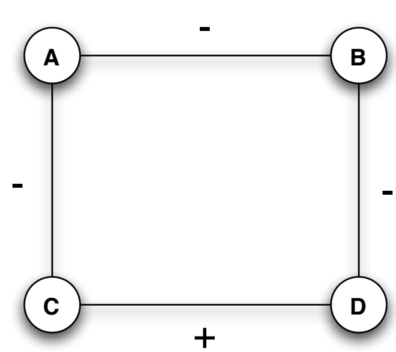

Figure 3: The graph for Question 3.

(3) Figure 3 depicts another graph with missing edges on which we can ask exactly the same question: is it possible to fill in the missing labeled edges in Figure 3 so as to get a labeled complete graph that is balanced? If so, describe the labels you would put on the missing edges. If not, provide an explanation for why there is no way to fill in the missing edges so as to end up with a balanced complete graph.

Part B

In Chapter 2, and the parts of lecture concerning it, we discussed various ways of modeling the idea that certain nodes or edges in a social network serve to connect portions of the network that are otherwise not well-connected. Such nodes and edges can play a kind of "gatekeeping" role, acting as the point of contact between these different parts of the network. To supplement these discussions, we now explore a set of definitions that are technically distinct from the ones in the book, but which also seek to address these ideas.

The first definition is the following: we say that a node X is a *gatekeeper* if for some other two nodes Y and Z, every path from Y to Z passes through X. For example, in the graph in Figure 4, node A is a gatekeeper, since it lies for example on every path from B to E. (It also lies on every path between other pairs of nodes — for example, the pair D and E, as well as other pairs.)

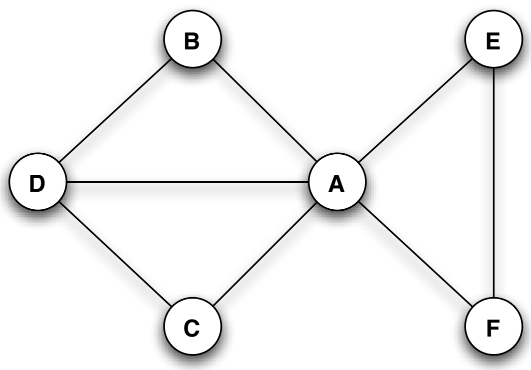

Figure 4: Node A is a gatekeeper. Node D is a local gatekeeper but not a gatekeeper.

This definition has a certain "global" flavor, since it requires that we think about paths in the full graph in order to decide whether a particular node is a gatekeeper. A more "local" version of this definition might involve only looking at the neighbors of a node. Here's a way to make this precise: we say that a node X is a *local gatekeeper* if there are two neighbors of X, say Y and Z, that are not connected by an edge. (That is, for X to be a local gatekeeper, there should be two nodes Y and Z so that Y and Z each have edges to X, but not to each other.) So for example, in Figure 4, node A is a local gatekeeper as well as being a gatekeeper; node D, on the other hand, is a local gatekeeper but not a gatekeeper. (Node D has neighbors B and C that are not connected by an edge; however, every pair of nodes — including B and C — can be connected by a path that does not go through D.)

So we have two new definitions: *gatekeeper*, and *local gatekeeper*. When faced with new mathematical definitions, a strategy that is often useful is to explore them first through examples, and then to assess them at a more general level and try to relate them to other ideas and definitions. Let's try this in the next few questions.

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(4) Give an example (together with an explanation) of a graph in which more than half of all nodes are gatekeepers.

(5) Give an example (together with an explanation) of a graph in which there are no gatekeepers, but in which every node is a local gatekeeper.

(6) Finally, an open-ended question. Write 1-2 paragraphs in which you address the following questions.

> *Are there connections — either at a formal or informal level — between the definitions of gatekeepers and local gatekeepers here, and definitions of related ideas in Chapter 2? In what sense, informally speaking, would you say that gatekeepers and local gatekeepers in a network, as defined here, actually play a "gatekeeping" role? In what ways do the definitions of gatekeepers and local gatekeepers fail to capture aspects of the related ideas from Chapter 2?*