| **Networks: Spring 2009** | **Practice Final Exam** |
|---|---|
| **David Easley and Jon Kleinberg** | **April 27, 2009** |

The final exam is **Wednesday, May 6, 7:00 - 9:30 PM in Barton Hall.** It will be a closed-notes exam. The best guide to the coverage of the exam is the contents of the course lectures; it will also be useful to review the homeworks and the readings.

To help in studying, we are providing the following practice final exam below. It is structured to approximately resemble the real final, although of course the actual questions on the real final may cover topics from the course that are not explicitly the focus of any question here. The practice final questions are not meant to be handed in; rather, we will discuss them at the final exam review sessions and during office hours.

Office hours will be held at their usual times up through the time of the final exam. The review sessions are Monday, May 4 at 2:00 PM in Goldwin Smith G64 and Tuesday, May 5 at 3:00 PM in Upson B17.

---

**(1)** Suppose we have a set of 3 sellers labeled $a$, $b$, and $c$, and a set of 3 buyers labeled $x$, $y$, and $z$. Each seller is offering a distinct house for sale, and the valuations of the buyers for the houses are as follows.

| Buyer | Value for $a$'s house | Value for $b$'s house | Value for $c$'s house |
|---|---|---|---|
| x | 5 | 7 | 1 |
| y | 2 | 3 | 1 |
| z | 5 | 4 | 4 |

Suppose that sellers $a$ and $b$ each charge 2, and seller $c$ charges 1. Is this set of prices market-clearing? Give a brief explanation.

**(2)** Suppose a search engine has two ad slots that it can sell. Slot $a$ has a clickthrough rate of 10 and slot $b$ has a clickthrough rate of 5. There are three advertisers who are interested in these slots. Advertiser $x$ values clicks at 3 per click, advertiser $y$ values clicks at 2 per click, and advertiser $z$ values clicks at 1 per click.

Compute the socially optimal allocation and the VCG prices for it. Give a brief explanation for your answer.

**(3)** In the payoff matrix below the rows correspond to player A's strategies and the columns correspond to player B's strategies. The first entry in each box is player A's payoff and the second entry is player B's payoff.

| | | Player B | |
|---|---|---|---|
| | | $x$ | $y$ |
| Player A | $x$ | $1, 1$ | $0, 1$ |
| | $y$ | $1, 0$ | $2, 2$ |

(a) Find all pure strategy Nash equilibria.

(b) Find all Evolutionarily Stable (pure) strategies.

(c) Briefly explain how the sets of predicted outcomes (in parts (a) and (b)) relate to each other.

**(4)** In the basic "six degrees of separation" question, one asks whether most pairs of people in the world are connected by a path of at most six edges in the social network, where an edge joins any two people who know each other on a first-name basis.

Now let's consider a variation on this question. Suppose that we consider the full population of the world, and suppose that from each person in the world we create a directed edge only to their ten closest friends (but not to anyone else they know on a first-name basis). In the resulting "closest-friend" version of the social network, is it possible that for each pair of people in the world, there is a path of at most six edges connecting this pair of people? Explain.

**(5)** Suppose that some researchers studying educational institutions decide to collect data to address the following two questions.

1. As a function of $k$, what fraction of Cornell classes have $k$ students enrolled?
2. As a function of $k$, what fraction of 3rd-grade elementary school classrooms in New York State have $k$ pupils?

Which one of these would you expect to more closely follow a power-law distribution as a function of $k$? Give a brief explanation for your answer, using some of the ideas about power-law distributions developed in class.

**(6)** Consider the model of the market for lemons from class. Suppose that there are three types of used cars: good ones, bad ones and lemons, and that sellers know which type of car they have. Buyers do not know which type of car a seller has. The fraction of used cars of each type is $\frac{1}{3}$ and buyers know this. Let's suppose that a seller who has a good car values it at \$8,000, a seller with a bad car values it at \$5,000 and a seller with a lemon values the lemon at \$1,000. A seller is willing to sell his car for any price greater than or equal to his value for the car; the seller is not willing to sell the car at a price below the value of the car. Buyers values for good cars, bad cars and lemons are, \$9,000, \$8,000 and \$4,000, respectively. As in class we will assume that buyers are risk-neutral; that is, they are willing to pay their expected value of a car.

(a) Is there an equilibrium in the used-car market in which all types of cars are sold? Explain briefly.

(b) Is there an equilibrium in the used-car market in which only bad quality cars and lemons are sold? Explain briefly.

(c) Is there an equilibrium in the used-car market in which only lemons are sold? Explain briefly.

**(7)** Consider the network depicted in Figure 1; suppose that each node starts with the behavior $B$, and each node has a threshold of $q = \frac{1}{2}$ for switching to behavior $A$.

**(a)** Now, let $e$ and $f$ form a two-node set $S$ of initial adopters of behavior $A$. If other nodes follow the threshold rule for choosing behaviors, which nodes will eventually switch to $A$?

**(b)** Find a cluster of density greater than $1 - q = \frac{1}{2}$ in $G - S$ that blocks behavior $A$ from spreading to all nodes, starting from $S$, at threshold $q$.

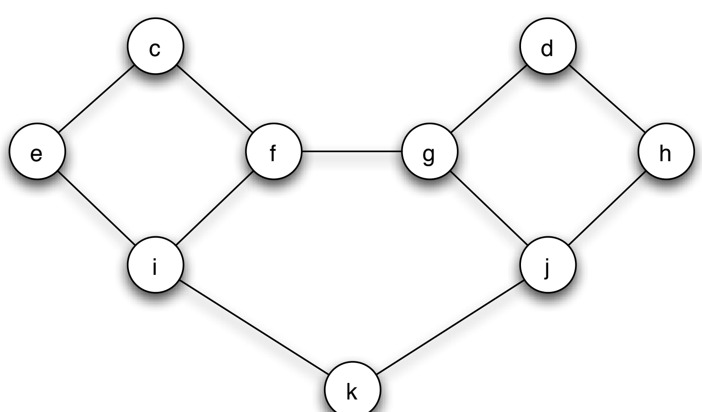


Figure 1:

**(8)** Show the values that you get if you run two rounds of computing hub and authority values on the network of Web pages in Figure 2. (That is, the values computed by the $k$-step hub-authority computation when we choose the number of steps $k$ to be 2.)

Show the values both before and after the final *normalization* step, in which we divide each authority score by the sum of all authority scores, and divide each hub score by the sum of all hub scores. It's fine to write the normalized scores as fractions rather than decimals.)

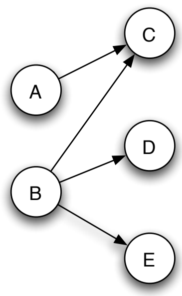


Figure 2:

**(9)** A seller will run a second-price, sealed-bid auction for an object. There are two bidders, $a$ and $b$, who have independent, private values $v_i$ which are either 0 or 1. For both bidders the probabilities of $v_i = 0$ and $v_i = 1$ are each 1/2. Both bidders understand the auction, but bidder $b$ sometimes makes a mistake about his value for the object. Half of the time his value is 1 and he is aware that it is 1; the other half of the time his value is 0 but occasionally he mistakenly believes that his value is 1. Let's suppose that when $b$'s value is 0 he acts as if it is 1 with probability $\frac{1}{2}$ and as if it is 0 with probability $\frac{1}{2}$. So in effect bidder $b$ sees value 0 with probability $\frac{1}{4}$ and value 1 with probability $\frac{3}{4}$. Bidder $a$ never makes mistakes about his value for the object, but he is aware of the mistakes that bidder $b$ makes. Both bidders bid optimally given their perceptions of the value of the object. Assume that if there is a tie at a bid of $x$ for the highest bid the winner is selected at random from among the highest bidders and the price is $x$.

(a) Is it bidding his true value still a dominant strategy for bidder $a$? Explain briefly

(b) What is the seller's expected revenue? Explain briefly.

**(10)** In this question we consider a variation on the model of information cascades from Chapter 15. Suppose that there is a new technology which individuals sequentially decide to adopt or reject. Let's suppose that anyone who adopts the new technology receives either a positive or a negative payoff from using the new technology. Unlike the model used in Chapter 15, these payoffs are random and have the property that the average payoff is positive if the technology is Good, and negative if the technology is Bad. Anyone who decides to reject the new technology always receives a payoff of exactly zero.

As in the model used in Chapter 15, each person receives a private signal about the technology and observes the actions of all those who chose previously. However, unlike the model used in Chapter 15, each person is also told the payoffs received by each of those who moved previously. [One interpretation of this is that a government agency collects information about individuals' experiences and distributes it for free as a public service.]

**(a)** Suppose that the new technology is actually Bad. How does this new information about payoffs (the payoffs received by each of those who moved previously) affect the potential for an information cascade of choices to adopt the new technology to form and persist? [You do not need to write a proof. A brief argument is sufficient.]

**(b)** Suppose that the new technology is actually Good. Can an information cascade of rejections of the new technology occur? Explain briefly.