Networks: Spring 2009	**Homework 3**
David Easley and Jon Kleinberg	**Due in Class February 20, 2009**

Homework will be due at the start of class on the due date. We cannot accept late homework except for University-approved excuses (which include illness with a note from Gannett, a family emergency, or travel as part of a University sports team or other University activity).

Parts A and B: Please hand in the the different parts separately, to facilitate grading. Please put your name and your netid on each part.

Reading: The questions below are primarily based on the material in Chapters 8-10 of the Networks book draft.

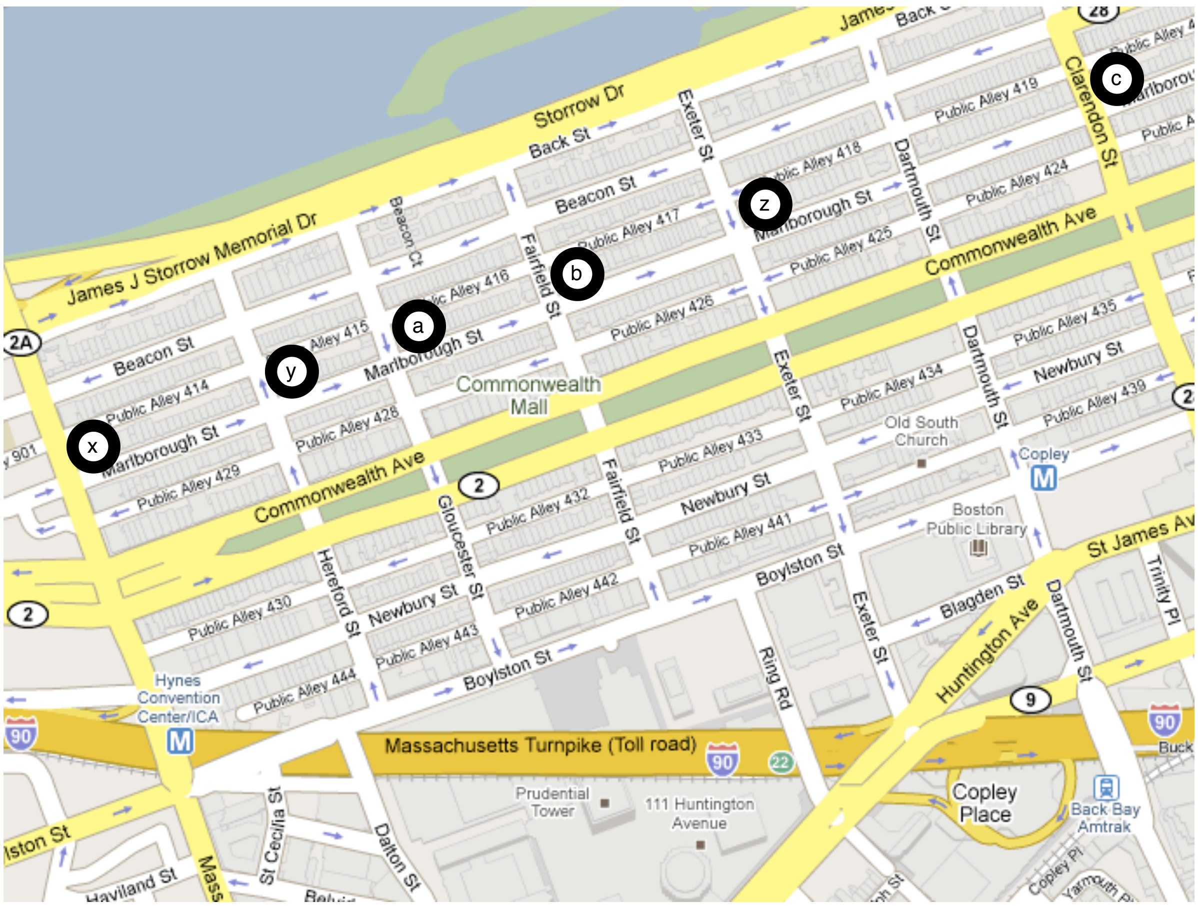

Figure 1: The graph for Question 1. (Image from Google Maps, http://maps.google.com/)

Part A

(1) Figure 1 shows a map of part of the Back Bay section of Boston. Suppose that the dark circles labeled x, y, and z represent people living in apartments in Back Bay who want to rent parking spaces by the month for parking their cars. (Due to the density of buildings, these parking spaces may be a short walk from where they live, rather than right at their apartment.) The dark circles labeled a, b, and c represent parking spaces available for rent.

Let's define the *distance* between a person and a parking space to be number of blocks they'd have to talk from their apartment to the parking space. Thus, for example, z is at a distance of 2 from space c, while y is at a distance of 5 from c and x is at a distance of 6 from c. (We'll ignore the fact that the block between Gloucester and Hereford is a bit shorter than the others; all blocks will be treated as the same in counting distance.)

Suppose that a person has a valuation for a potential parking space equal to

$$8 - \text{(their distance to the parking space)}.$$

(Notice that this formula gives higher valuations to closer parking spaces.) In terms of these valuations, we'd like to think about prices that could be charged for the parking spaces.

(a) Describe how you would set up this question as a matching market in the style of Chapter 9. Say who the sellers and buyers would be in your set-up, as well as the valuation each buyer has for the item offered by each seller.

(b) Describe what happens if we run the bipartite graph auction procedure from Chapter 9 on the matching market you set up in (a), by saying what the prices are at the end of each round of the auction, including what the final market-clearing prices are when the auction comes to an end.

(Note: In some rounds, you may notice that there are multiple choices for the constricted set of buyers. Under the rules of the auction, you can choose any such constricted set. It's interesting to consider — though not necessary for this question — how the eventual set of market-clearing prices depends on how one chooses among the possible constricted sets.)

(c) At a more informal level, how do the prices you determined for the parking spaces in (b) relate to these spaces' intuitive "attractiveness" to the people in apartments x, y, and z? Explain.

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(2) Suppose we have a set of 2 sellers labeled a and b, and a set of 2 buyers labeled x and y. Each seller is offering a distinct house for sale, and the valuations of the buyers for the houses are as follows.

Buyer	Value for a's house	Value for b's house
x	4	1
y	3	2

In general, there will be multiple sets of market-clearing prices for a given set of sellers, buyers, and valuations: any set of prices that produces a preferred-seller graph with a perfect matching is market-clearing.

As a way of exploring this issue in the context of the example above, give three *different* sets of market-clearing prices for this matching market. The prices should be whole numbers (i.e. they should be numbers from $0, 1, 2, 3, 4, 5, 6, \ldots$). (Note that for two sets of market-clearing prices to be different, it is enough that they not consist of exactly the same set of numbers.) Explain your answer.

Part B

(3) In this problem we will examine a second-price, sealed-bid auction. Assume that there are two bidders who have independent, private values v_i which are either 1 or 7. For each bidder, the probabilities of $v_i = 1$ and $v_i = 7$ are each 1/2. So there are four possible pairs of the bidders' values (v_1, v_2): $(1,1), (1,7), (7,1)$, and $(7,7)$. Each pair of values has probability 1/4.

Assume that if there is a tie at a bid of x for the highest bid the winner is selected uniformly at random from among the highest bidders and the price is x.

(a) For each pair of values what will bid will each bidder submit, what price will the winning bidder pay, and how much profit (the difference between the winning bidder's value and price he pays) will the winning bidder earn?

(b) Now let's examine how much revenue the seller can expect to earn and how much profit the bidders can expect to make in the second price auction. Both revenue and profit depend on the values, so let's calculate the average of each of these numbers across all four of the possible pairs of values. [Note that in doing this we are computing each bidder's expected profit before the bidder knows his value for the object.] What is the seller's expected revenue in the second price auction? What is the expect profit for each bidder?

(c) The seller now decides to charge an entry fee of 1. Any bidder who wants to participate in the auction must pay this fee to the seller before bidding begins and, in fact, this fee is imposed before the each bidder knows his or her own value for the object. The bidders know only the distribution of values and that anyone who pays the fee will be allowed to participate in a second price auction for the object. This adds a new first stage to the game in which bidders decide simultaneously whether to pay the fee and enter the auction, or to not pay the fee and stay out of the auction. This first stage is then followed by a second stage in which anyone who pays the fee participates in the auction. We will assume that after the first stage is over both potential bidders learn their own value for the object (but not the other potential bidder's value for the object) and that they both learn whether or not the other potential bidder decided to enter the auction.

Let's assume that any potential bidder who does not participate in the auction has a profit of 0, if no one chooses to participate then the seller keeps the object and does not run an auction, if only one bidder chooses to participate in the auction then the seller runs a second price auction with only this one bidder (and treats the second highest bid as 0), and finally if both bidders participate the second price auction is the one you solved in part (a).

Is there an equilibrium in which each bidder pays the fee and participates in the auction? Give an explanation for your answer.

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(4) A seller announces that he will sell a case of rare wine using a sealed-bid, second-price auction. A group of I individuals plan to bid on this case of wine. Each bidder is interested in the wine for his or her personal consumption; the bidders' consumption values for the wine may differ, but they don't plan to resell the wine. So we will view their values for the wine as independent, private values (as in Chapter 8 of the text). You are one of these bidders; in particular, you are bidder number i and your value for the wine is v_i.

How should you bid in each of the following situations? In each case, provide an explanation for your answer; a formal proof is not necessary.

(a) You know that a group of the bidders will collude on bids. This group will chose one bidder to submit a "real bid" of v and the others will all submit bids of 0. You are not a member of this collusive group and you cannot collude with any other bidder.

(b) You, and all of the other bidders, have just learned that this seller will collect bids, but won't actually sell the wine according the rules of a second-price auction. Instead, after collecting the bids the seller will tell all of the bidders that some other fictional bidder actually submitted the highest bid and so won the auction. This bidder, of course, doesn't exist so the seller will still have the wine after the auction is over. The seller plans to privately contact the highest actual bidder and tell him or her that the fictional high bidder defaulted (he didn't buy the wine after all) and that this bidder can buy the wine for the price he or she bid in the auction. You cannot collude with any bidder. [You do not need to derive an optimal bidding strategy. It is enough to explain whether your bid would differ from your value and if so in what direction.]

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(5) Consider a trading network with intermediaries in which there are three sellers $S1, S2, S3$, two buyers $B1, B2$, and two traders (intermediaries) $T1, T2$. Sellers $S1$ and $S2$ can trade only with trader $T1$; and, seller $S3$ can trade only with trader $T2$. The buyers can each trade with only one of the traders: buyer $B1$ can only trade with trader $T1$; and buyer $B2$ can only trade with trader $T2$. The sellers each has one unit of the object and value it at 0. The buyers are not endowed with the object and they each value a unit at 1.

(a) Draw the trading network, with the traders as squares, the buyers and sellers as circles, and with edges connecting nodes who are able to trade with each other. Label each node as $S1$, $S2$, $S3$, $B1$, $B2$, $T1$ or $T2$.

(b) Describe what the possible Nash equilibria are, including both prices and the flow of goods. Give an explanation for your answer.

(c) Suppose now that we add an edge between buyer $B2$ and trader $T1$. We want to examine whether this new edge changes the outcome in the game. To do this, take the equilibrium from your answer to (b), keep the prices and good flows on the edges from (b) the same as before, and then suppose that the ask price on the new $B2$-$T1$ edge is 1, and there is no good flow on this new edge. Do these prices and good flows still form an equilibrium? If you think that the answer is yes, give a brief (1-3 sentence) explanation why. If you think the answer is no, describe a way in which one of the participants in the game would deviate.