Networks: Spring 2009	Homework 4
David Easley and Jon Kleinberg	Due in Class March 27, 2009

Homework will be due at the start of class on the due date. We cannot accept late homework except for University-approved excuses (which include illness with a note from Gannett, a family emergency, or travel as part of a University sports team or other University activity).

NetID: Please write your NetID on each part of your homework.

Parts A and B: Please hand in the the different parts separately, to facilitate grading.

Reading: The questions below are primarily based on the material in Chapters 12-14 of the Networks book draft.

Part A

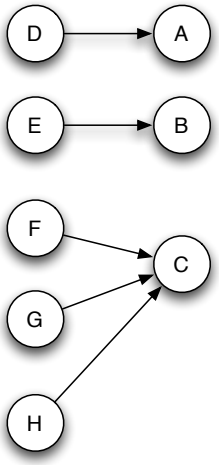

Figure 1: The network of Web pages for Question (1a).

(1) (a) Show the values that you get if you run two rounds of computing hub and authority values on the network of Web pages in Figure 1. (That is, the values computed by the k-step hub-authority computation when we choose the number of steps k to be 2.)

Show the values both before and after the final *normalization* step, in which we divide each authority score by the sum of all authority scores, and divide each hub score by the sum of all hub scores. (We will call the scores obtained after this dividing-down step the *normalized scores*. It's fine to write the normalized scores as fractions rather than decimals.)

(b) Due to the symmetry of nodes A and B in part (a), you should have seen that they get the same authority scores. Now let's look at what happens to the scores when node E,

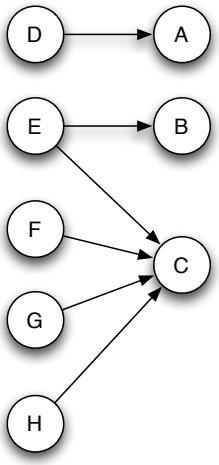

Figure 2: The network of Web pages for Question (1b).

which links to B, decides to link to C as well. This produces the new network of Web pages shown in Figure 2.

Similarly to part (a), show the normalized hub and authority values that each node gets when you run the 2-step hub-authority computation on the new network in Figure 2.

(c) In (b), which of nodes A or B now has the higher authority score? Give a brief explanation in which you provide some intuition for why the difference in authority scores between A and B in (b) turned out the way it did.

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(2) Let's consider the limiting values that result from the Basic PageRank Update Rule (i.e. the version where we don't introduce a scaling factor s). In Chapter 13, these limiting values are described as capturing "a kind of equilibrium based on direct endorsement: they are values that remain unchanged when everyone divides up their PageRank and passes it forward across their out-going links."

This description gives a way to check whether an assignment of numbers to a set of Web pages forms an equilibrium set of PageRank values: the numbers should add up to 1, and they should remain unchanged when we apply the Basic PageRank Update Rule. For example, this is illustrated in the book via Figure 13.6: you can check that if we assign a PageRank of $4/13$ to page A, $2/13$ to each of B and C, and $1/13$ to the five other pages, then these numbers add up to 1 and they remain unchanged when we apply the Basic PageRank Update Rule. Hence they form an equilibrium set of PageRank values.

For each of the following two networks, use this approach to check whether the numbers indicated in the figure form an equilibrium set of PageRank values. (In cases where the numbers do not form an equilibrium set of PageRank values, you do not need to give numbers that do; you simply need to explain why the given numbers do not.)

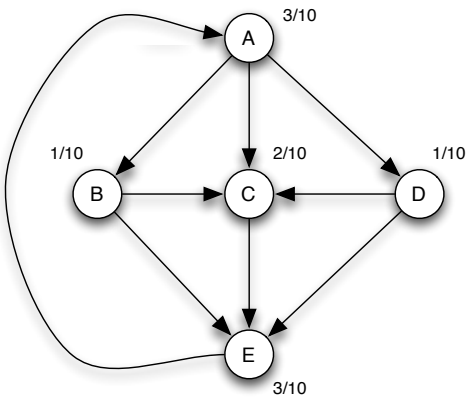

Figure 3: The network of Web pages for Question (2a).

(a) Does the assignment of numbers to the nodes in Figure 3 form an equilibrium set of PageRank values for this network of Web pages? Give an explanation for your answer.

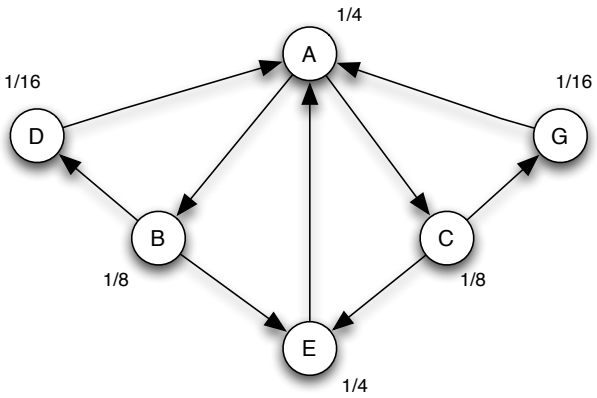

Figure 4: The network of Web pages for Question (2b).

(b) Does the assignment of numbers to the nodes in Figure 4 form an equilibrium set of PageRank values for this network of Web pages? Give an explanation for your answer.

Part B

(3) The book discusses the relationship between the VCG Principle and second price auctions. In particular, we saw that the VCG Principle is a generalization of the idea behind second price auctions to a setting in which there is more than one object being sold. In this problem we will explore this relationship in an example. Suppose that a seller has one item,

which we'll call item x. There are three buyers, whom we'll call a, b, and c. The values that these buyers (a, b, and c) have for the item are 6, 3, and 1, respectively.

(a) Suppose that the seller runs a second price auction for the item. Which buyer will win the auction and how much will this buyer pay?

(b) Now let's suppose that the seller uses the VCG procedure to allocate the item. Remember that the first step in the running the VCG procedure when there are more buyers than items is to create fictional items, which each buyer values at 0, so that the number of items to be allocated is the same as the number of bidders. Let's call these additional (fictional) items y and z. Find the allocation that results from running the VCG procedure. What are the prices charged to each buyer for the item that they receive? Explain why the price that buyer a pays is the harm that he causes to the remaining bidders by taking the item he is assigned.

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(4) Suppose a search engine has two ad slots that it can sell. Slot a has a clickthrough rate of 4 and slot b has a clickthrough rate of 3. There are three advertisers who are interested in these slots. Advertiser x values clicks at 4 per click, advertiser y values clicks at 3 per click, and advertiser z values clicks at 1 per click.

(a) Suppose that the search engine runs the VCG Procedure to allocate slots. What assignment of slots will occur and what prices will the advertisers pay? Give an explanation for your answer.

(b) Now the search engine is considering the creation of a third ad slot which will have a clickthrough rate of 2. Let's call this new ad slot c. Suppose that search engine does create this slot and again uses the VCG Procedure to allocate slots. What assignment of slots will occur and what prices will the advertisers pay? Give an explanation for your answer.

(c) What revenue will the search engine receive from the VCG Procedure in parts (a) and (b)? If you were running the search engine, given this set of advertisers and slots, and could choose whether to create slot c or not, what would you do? Why? (In answering this question assume that you have to use the VCG Procedure to allocate any slots you create.)