Networks: Spring 2009	Homework 5
David Easley and Jon Kleinberg	Due in Class April 10, 2009

Homework will be due at the start of class on the due date. We cannot accept late homework except for University-approved excuses (which include illness with a note from Gannett, a family emergency, or travel as part of a University sports team or other University activity).

NetID: Please write your NetID on each part of your homework.

Parts A and B: Please hand in the the different parts separately, to facilitate grading.

Reading: The questions below are primarily based on the material in Chapters 15 and 16 of the Networks book draft.

Part A

(1) Lets consider the model of information cascades. Assume that the probability that the state is Good (G) is $p = 1/2$, and that the probability of a High signal given a Good state is $q = 2/3$. (The probability of a Low signal given a Bad state is also $q = 2/3$.) Remember that each person observes a signal and the choices (but not the signals) of all those who chose before him. Each person chooses between Accept (A) and Reject (R). Reject has a payoff of 0 and Accept has a positive payoff if the state is Good and a negative payoff if the state is Bad. In each of the parts below show your work.

(a) What is the probability that the first person to decide will choose R is if the state is G? What is the probability that the first person to decide will choose R if the state is B?

(b) Suppose that the state is G. (i) What is the probability that the first two people to decide will both choose R? (ii) What is the probability that they will both choose A? (iii) What are these two probabilities if the state is B?

(c) What is the probability of a cascade on a wrong choice (R if the state is G, or A if the state is B) beginning with the decision by the third person? What is the probability of a cascade on a right choice (A if the state is G, or R if the state is B) beginning with the decision by the third person?

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(2) In this problem we will consider a good with a network effect. There are 200 potential purchasers numbered from 1 to 200. Individual n has reservation price $r(n) = 200 - n$ before we consider the network effect. The network effect is given by $f(m) = m/100$ for m users. So the relationship between the number of users m and the willingness to pay of user m is $r(m)f(m) = 2m - m^2/100$. In each of the parts below show your work.

(a) Suppose that the price of the good is 100. Find all the equilibrium values of m.

(b) Now let's suppose that the network effect becomes more important and so now $f(m) = m/75$. The price is still 100. Find all of the new equilibrium values of m.

Part B

(3) Lets return to the cascade environment setup from Problem (1) in Part A. The probabilities are again $p = 1/2$ and $q = 2/3$. Suppose that you are the tenth person to make a choice and you have observed that everyone before you chose R. That is, we are in an R-cascade.

In this problem (as we did in class and in the text) we assume that no one makes a mistake. That is, everyone makes an optimal choice given whatever information they have. We will also assume that if someone can observe or infer an equal number of High and Low signals then they will follow their own signal.

(a) What is the probability that this is an incorrect cascade? [The probability that the state is actually G given that we are in this R-cascade.]

(b) Now let's suppose that before you (person 10) receive your signal, you decide to ask person 9 about the signal that they observed. Let's suppose that person 9 observed a High signal, that person 9 tells you that his signal was High, and that you know that person 9 is telling the truth. After this, you receive your own signal. What decision should you make, A or R, and how does it depend on which signal you receive?

(c) Now let's consider person 11. Person 11 observes only his own signal and the choices of those who decided before him (1 to 10). Person 11 knows that you have observed both your signal and person 9's signal. Person 11 cannot observe these signals; all he knows is the choices that have been made. The first nine people have chosen R. What should person 11 do if you choose R? What should he do if you choose A? Why? Remember that person 11 observes a signal, and so his choice can depend on his signal as well as the earlier choices.

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(4) In lecture, and in the book chapter on network effects, we focused on goods with positive network effects: ones for which additional users made the good more attractive for everyone. But we know from our earlier discussion of Braess's Paradox that network effects can sometimes be negative: more users can sometimes make an alternative less attractive, rather than more attractive. Some goods actually have both effects. That is, the good may become more attractive as more people use it as long there aren't too many users, and then once there are too many users it becomes less attractive as more people use it. Think of a club in which being a member is more desirable if there is a reasonable number of other members, but once the number of members gets too large the club begins to seem crowded and less attractive. Here we explore how our model of network effects can incorporate such a combination of effects.

In keeping with the notation in the book let's assume that there are 200 potential users numbered from 1 to 200. Individual n has the reservation price $r(n) = 200 - n$ before we consider the network effect. The network effect is given by $f(m) = m$ for $m \leq 50$ and by $f(m) = 100 - m$ for $m \geq 50$. So the network benefit to being a user is maximized when there are 50 users, once there are more than 50 users the network benefit declines, and it becomes negative if there are more than 100 users. Suppose that the price of this good is p where $0 < p < 7500$.

(a) How many equilibria are there? Why? [You do not need to solve for the number(s) of users; a graph and explanation is fine.]

(b) Which equilibria are stable? Why?

(c) Consider an equilibrium in which someone is using the good. Is social welfare maximized at this number of users, or would it go up if there were more users, or would it go up if there were fewer users? Explain. [Again no calculations are necessary; a careful explanation is sufficient.]