Networks: Spring 2009	Homework 6
David Easley and Jon Kleinberg	Due in Class April 24, 2009

Homework will be due at the start of class on the due date. We cannot accept late homework except for University-approved excuses (which include illness with a note from Gannett, a family emergency, or travel as part of a University sports team or other University activity).

NetID: Please write your NetID on each part of your homework.

Parts A and B: Please hand in the the different parts separately, to facilitate grading.

Reading: The questions below are primarily based on the material in Chapters 6, 18, and 19 of the Networks book draft.

Part A

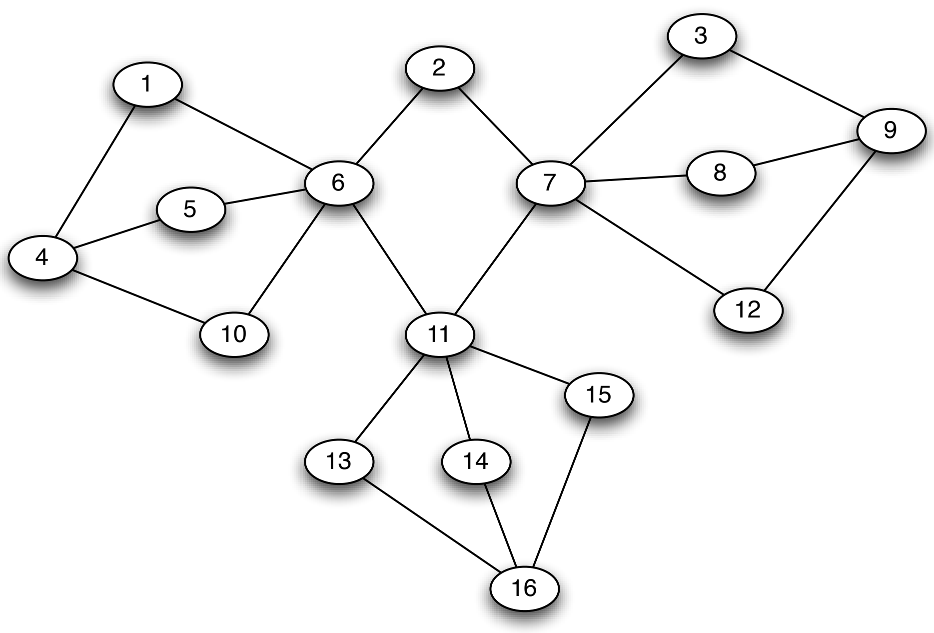

Figure 1: The social network for Question 1.

(1) Consider the model we've discussed in class and in Chapter 19 for the diffusion of a new behavior through a social network.

Suppose that initially everyone is using behavior B in the social network in Figure 1, and then a new behavior A is introduced. This behavior has a threshold of $q = 1/2$: any node will switch to A if at least $1/2$ of its neighbors are using it.

(a) Find a set of three nodes in the network with the property that if they act as the three initial adopters of A, then it will spread to all nodes. (In other words, three nodes who are capable of causing a cascade of adoptions of A.)

(b) Is the set of three nodes you found in (a) the only set of three initial adopters capable of causing a cascade of A, or can you find a different set of three initial adopters who could also cause a cascade of A?

(c) Find three clusters in the network, each of density greater than 1/2, with the property that no node belongs to more than one of these clusters.

(d) How does your answer to (c) help explain why there is no set consisting of only two nodes in the network that would be capable of causing a cascade of adoptions of A? (I.e., only two nodes that could cause the entire network to adopt A.)

Part B

(2) In this problem we will consider the relationship between Nash equilibria and evolutionarily stable strategies for games with a strictly dominant strategy. First, let's define what we mean by *strictly dominant*. In a two-player game, strategy X is said to be a strictly dominant strategy for a player i if, no matter what strategy the other player j uses, player i's payoff from using strategy X is strictly greater than his payoff from any other strategy. Consider the following game in which a, b, c, and d are non-negative numbers.

		Player B	
		X	Y
Player A	X	a, a	b, c
	Y	c, b	d, d

Suppose that strategy X is a strictly dominant strategy for each player, i.e. $a > c$ and $b > d$.

(a) Find all of the pure strategy Nash equilibria of this game.

(b) Find all of the evolutionarily stable strategies of this game.

(c) How would your answers to parts (a) and (b) change if we change the assumption on payoffs to: $a > c$ and $b = d$?

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(3) In this problem we are going to examine how a tax on the purchase of used cars might affect the price and quantity of used cars traded. Suppose that there are two types of used cars: good ones and bad ones. Sellers of used cars know the type of car that they own. Buyers do not know which type of car any particular seller has. Buyers do know that there are good and bad used cars, and they know that of the people who own used cars and are interested in selling their car, 1/2 have good cars and 1/2 have bad cars. Let's suppose that there are more possible buyers of used cars than there are possible sellers of used cars. [As in class we want to assume that there are more buyers than sellers to make the analysis straightforward.] A seller who has a good used car values it at $8,000 and a seller who has a bad used car values it at $3,000. A seller is willing to sell his car for any price greater than or equal to his value for the car; no seller is willing to sell his car for a price less than his value for the car. Buyers values for good and bad used cars are $10,000 and $6,000, respectively. As in class we will assume that buyers each want at most one used car and they are willing to pay their expected value for a used car.

(a) Find all of the equilibria in the market for used cars. For each equilibrium provide the price of used cars and the number of used cars traded.

(b) Now suppose that the government places a tax of $100 on the purchase of used cars. That is, anyone who buys a used car must pay a tax of $100 on the purchase of the car. This effectively lowers the value that buyers place on any type of used car by $100. Find all of the equilibria in the market for used cars.

(c) Now let's change the setup of the problem a bit so that there are three types of used cars: good ones, bad ones and lemons. Now 1/3 of the possible sellers have good cars, 1/3 have bad cars and 1/3 have lemons. Buyers and sellers values for good and bad used cars are the same as before. Everyone (both buyers and sellers) values a lemon at 0. There are still more buyers than sellers.

i. There is no tax on the purchase of used cars. Find all of the equilibria in the market for used cars.

ii. Now the government imposes a tax of $100 on the purchase of used cars. Find all of the equilibria in the market for used cars.